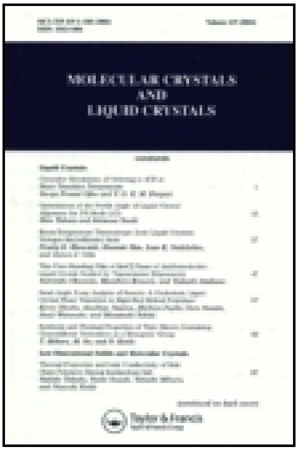

Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl19

Propagation of Optical Fields in a Planar Liquid Crystal Waveguide

R. F. Rodriguez [a] & J. A. Reyes [a]
[a] Departamento de Física-Química, Institute de Fisica, U.N.A.M. Apdo. Postal 20-364, 01000 México, D.F., México
Version of record first published: 24 Sep 2006.

To cite this article: R. F. Rodriguez & J. A. Reyes (1996): Propagation of Optical Fields in a Planar Liquid Crystal Waveguide, Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals, 282:1, 287-296

To link to this article: http://dx.doi.org/10.1080/10587259608037582

Mol. Cryst. Liq. Cryst., 1996, Vol. 282, pp. 287–296
Reprints available directly from the publisher
Photocopying permitted by license only

PROPAGATION OF OPTICAL FIELDS IN A PLANAR LIQUID CRYSTAL WAVE GUIDE

R. F. RODRIGUEZ and J. A. REYES
Departamento de Física-Química, Instituto de Física, U.N.A.M.
Apdo. Postal 20-364, 01000 México, D.F., México

Abstract A model for the propagation of an optical field in a planar nematic cell is presented. The effects produced by the hydrodynamic couplings due to backflows and to an externally induced flow are explicitly taken into account. It is shown that when the resulting flow is a planar Couette flow, the liquid crystal distortion induced by the field gives rise to an index-gradient which generates a wave guiding effect that concentrates the ray trajectories and the electromagnetic energy density into the central part of the cell.

Keywords: wave guides, nematic cell, hydrodynamic effects

INTRODUCTION

The propagation of a laser beam through a nematic liquid crystal film is a phenomenon that exhibits some very unique and highly nonlinear optical properties.[1,2] It has been experimentally verified that a sufficiently strong laser field induces an orientational transition in a nematic film.[3] For instance, a linearly polarized incident beam distorts the initial alignment in the film by reorienting the molecules against the elastic torques. On the other hand, when the beam is circularly or elliptically polarized, a variety of nonlinear dynamical regimes may arise during the reorientation process.[4]

In recent years a great deal of attention has been given to a different aspect of this process, namely, the possibility of producing a wave guiding effect in optical fibers with liquid crystalline cores.[3-6] The basic idea is to take profit of the nonlinear optical properties of liquid crystals to produce the effect without restoring to the usual mechanism based on total internal reflection. In a previous paper[7] (hereafter refered to as I) we have modeled this situation and we have shown the existence of a wave guiding effect that concentrates the ray trajectories and the electromagnetic energy of the beam in the central part of the cell. However, in I all the hydrodynamic effects that usually accompany the reorientation process were entirely neglected. The basic purpose of this work is to extend the analysis in I to take into account the combined action of backflows and of externally induced hydrodynamic flows on the reorientation. We show that for a low intensity beam in the optical and WKB limits, the wave guiding effect still exists. In the former case this is exhibited by calculating the ray trajectories from the corresponding eikonal equation; while in the latter one it is shown by calculating the spatial distribution of the electromagnetic energy density within the cell. In both cases there exist caustics whose position is determined by the angle of incidence and the material parameters.

We consider a nematic layer of thickness l measured along the z axis and contained between two parallel conducting plates, as shown in Fig. 1.The transverse dimensions along the x and y directions are large compared to l, but the cell has a finite volume $V = L^2 l$. We assume strong anchoring boundary conditions for the director at the plates. If the nematic is excited by an obliquely incident laser beam whose polarization always remains in the x-z plane, we shall assume that the reorientation of $\vec{n}$ will also take place in the x-z plane. Furthermore, if the cell has a large aspect ratio, we may assume spatial homogenity in the x direction so that $\vec{n} = [\sin \theta(z,t), 0, \cos \theta(z,t)]$. The reorientation angle θ satisfies the strong anchoring planar boundary conditions $\theta(z=\pm l/2) = \pm\pi/2$.

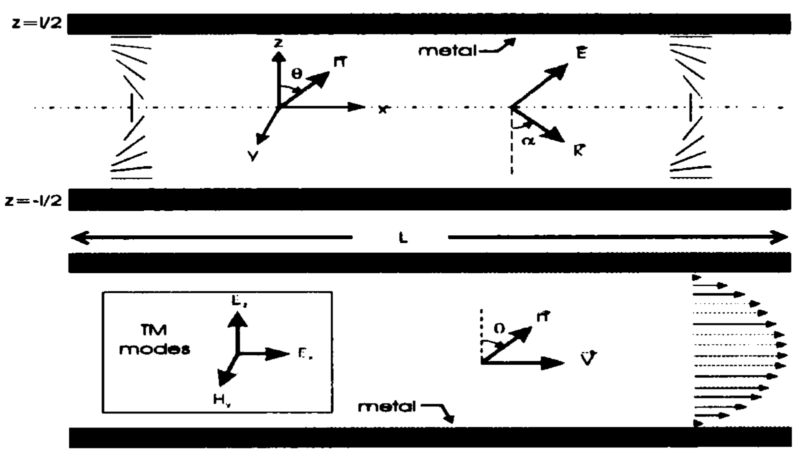

FIGURE 1 Schematics of a linearly polarized laser beam propagating through a planarly aligned liquid crystal film.

BASIC DYNAMICAL EQUATIONS

For the assumed geometry and in the approximation of equal elastic constants ($k \equiv k_{11} = k_{22} = k_{33}$), the Helmholtz free energy functional is

$$\mathcal{F} = \int_V dv\ \{(1/2)k\ (d\theta/dz)^2 + (1/8\pi)[\varepsilon_\perp|E|^2 + \varepsilon_a(|E_x|^2\sin^2\theta +$$

$$(E_x^* E_z + E_x E_z^*)\sin\theta\ \cos\theta + |E_z|^2\cos^2\theta) + H_y^2 + (1/2)\rho v_x(z)^2\}, \quad (1)$$

where $\rho(z,t)$ is the mass density and * denotes complex conjugate; $\varepsilon_a \equiv \varepsilon_\parallel - \varepsilon_\perp$ stands for the dielectric anisotropy of the nematic. In writing this expression we have assumed that the induced backflows also occur in the x-z plane and that due to the features of the cell, their only relevant component is along the x axis. The stationary configurations are determined by minimizing (1) and are defined by the corresponding Euler-Lagrange equations

$$\delta\mathcal{F}/\delta\theta \equiv k\ d^2\theta/dz^2 + (\varepsilon_a/8\pi)[\sin 2\theta\ (|E_x|^2 - |E_z|^2) -$$

$$- 2E_x E_z^* \cos 2\theta] = 0. \quad (2)$$

The equations of motion for the reorientation angle and the velocity field can be obtained from $\mathcal{F}$ [8] and turn out to be

$$\partial\theta/\partial t = - (2/\gamma_1)\ \delta\mathcal{F}/\delta\theta - (\lambda-1)\cos\theta\ \partial v_x/\partial z, \quad (3a)$$

$$\partial v_x/\partial t = (\nu_3/\rho)\ \partial^2 v_x/\partial z^2 + \{(\lambda-1)/2\rho\}\ \partial[(\cos\theta)^{-1}\ \delta\mathcal{F}/\delta\theta]/\partial z -$$

$$- \rho^{-1}\ dp/dx \quad (3b)$$

where γ_1, γ_2, ν_3 with $\lambda = \gamma_1/\gamma_2$, denote various viscosity coefficients. The last term in Eq.(3b) can not be derived from $\mathcal{F}$ and takes into account the contribution to the flow produced by an external pressure gradient along the x direction. We assume that the total field v_x satisfies stick boundary conditions on the plates, $v_x(z=\pm l/2)$. The coupled equations (3) describe the time evolution of the orientational configuration induced by the beam in the presence of hydrodynamic flow.

As shown in I, the propagation of the beam may be conveniently described in terms of the complete representation provided by the corresponding TM modes $E_x(\zeta,k_0)$, $E_z(\zeta,k_0)$ and $H_y(\zeta,k_0)$ (see Fig.1), where $\zeta \equiv z/l$ and $k_0 \equiv \omega/c$ is the free space wavenumber, being c the speed of light in vacuum. These modes are the only ones that couple to the reorientational dynamics and are assumed to be of the general form

$$\mathcal{G}_i(x,z,t) = G_i(\zeta,k_0)\exp[i(\beta x-\omega t)],\quad i = x,\ z,\ y, \tag{4}$$

where β is the propagation constant. Using the Maxwell equations without sources in the liquid crystal, one easily arrives at the following set of equations for the amplitudes G_i of these modes:

$$\varepsilon_{zz} d H_y/d\zeta + [2ik_0 lp\varepsilon_{xz} + d\varepsilon_{zz}/d\zeta]\, dH_y/d\zeta + k_0 l[k_0 l(\varepsilon_{\parallel}\varepsilon_{\perp} - p^2\varepsilon_{xx}) + $$

$$+ ipd\varepsilon_{xz}/d\zeta]\, H_y = 0\ , \tag{5a}$$

$$E_x = (1/\varepsilon_{\parallel}\varepsilon_{\perp})\ [-\ \varepsilon_{xz}p\ H_y + (i/k_0 l)\varepsilon_{zz} dH\ /d\zeta]\ , \tag{5b}$$

$$E_z = (1/\varepsilon_{\parallel}\varepsilon_{\perp})\ [\ p\varepsilon_{xx}\ H_y - (i/k_0 l)\varepsilon_{xz} dH_y/d\zeta]\ , \tag{5c}$$

with $p \equiv \beta/k_0$. The components of the dielectric tensor $\varepsilon_{ij} = \varepsilon_{\perp}\delta_{ij} + \varepsilon_a n_i n_j$ are explicity given in terms of $\theta(z)$ and the dielectric anisotropy by $\varepsilon_{xx} = \varepsilon_{\perp} + \varepsilon_a \sin^2\theta(z)$, $\varepsilon_{xz} = \varepsilon_a \sin\theta(z)\cos\theta(z)$, $\varepsilon_{zz} = \varepsilon_{\perp} + \varepsilon_a\cos^2\theta(z)$, $\varepsilon_{yy} = \varepsilon_{\perp}$ with $\varepsilon_{yx} = \varepsilon_{zy} = 0$.

REORIENTATION IN THE STATIONARY REGIME

The stationary state is defined by $\partial\theta/\partial t = 0$ and $\partial v/\partial t = 0$. If $dp/dx \equiv \Delta p/L$ = const, from Eqs. (3) one arrives at the closed equation

$$d^2v_x/dz^2 = \bar{v}_x/l^2, \tag{6}$$

where $\bar{v}_x \equiv Q/\rho lL$ is the average velocity and Q is the mass rate per unit time; when Q is calculated explicitly this quantity can be rewritten as $\bar{v}_x = (l^2\Delta p/2L)[\nu_3 + (\lambda-1)^2\gamma_1/4]$. For stick boundary conditions, the solution of (6) reads

$$v_x(\zeta) = 6\ \bar{v}_x(1/4 - \zeta^2), \tag{7}$$

and represents the velocity profile of the well known plane Couette flow. Substitution of this profile into Eq.(3a) leads then to the following nonlinear reorientational equation for the stationary state

$$\mathcal{L}\theta \equiv d^2\theta/d\zeta^2 + q[(|\bar{E}_z|^2 - |\bar{E}_x|^2)\sin 2\theta + (\bar{E}_x\bar{E}_z^* + \bar{E}_x^*\ \bar{E}_z)\cos 2\theta] +$$

$$6\ N\ \zeta\cos\theta = 0. \tag{8}$$

Here $\bar{E}_z \equiv E_z/E_0$ and $\bar{E}_x \equiv E_x/E_0$, where $E_0^2/8\pi$ is the intensity of the incident beam. There are two important and independent parameters in the above equation. On the one hand, the parameter $q \equiv \varepsilon_a E_0 l^2/8\pi k$, which is proportional to the ratio between the intensity of the optical beam and the nematic's elastic energy density, measures the strenght of the coupling between the optical field and the orientational configuration. On the other hand, the parameter $N \equiv 6\ R\ \nu_3(\lambda-1)\gamma_1/k\rho$ contains the effects due to the hydrodynamic flows. It depends on several material parameters and on the induced flow through the Reynolds number $R \equiv \rho\ \bar{v}_x l/\nu_3$. To solve Eq.(8) to a given order in q, the corresponding fields should be determined first from Eqs.(5) to order q-1. Since the components of ε_{ij} depend on θ, this requires to calculate θ also to order q-1. We start with the director configuration that minimizes the free energy in the abscence of optical fields (q = 0); this approximation defines the low intensity limit. However, even in this approximation Eq.(8) can not be solved exactly in an analytical form. But by minimizing a global error functional constructed with the operator $\mathcal{L}$ in Eq.(8), we arrive at the solution

$$\theta^{(0)}(\zeta) = C\zeta(\zeta^2 - 1/4) + \pi\zeta + O(q), \tag{9}$$

where C = N/3 for N>>1 and C = 2N/3 for N<1. For a Reynolds number $R = 10^{-2}$, which corresponds to N = 40, this configuration is shown in Fig. 2. Once this stationary state has been determined, the TM modes can be calculated from Eqs:(5) to zero order in q. As will be shown below, these modes will be used to determine the electromagnetic energy density within the cell. This process of succesive approximations can be carried on to higher orders in q. For instance, to calcluate the TM modes to first order in q, one should first calculate the change in the dielectric tensor due to the torques resulting from the optical fields by solving (8) up to order q to obtain $\theta^{(1)}(\zeta)$; in this case the fields to zero order in q should be used in Eq.(8). Once $\theta^{(1)}(\zeta)$ is known, from Eqs.(5) the fields to first order in q can be calculated. In this sense both, the reorientational equation (8) and Eqs.(5) for the field amplitudes are nonlinear. However, in this work we only solve Eq.(8) in the low intensity limit q<<1.

OPTICAL LIMIT

This limit is defined by the condition $k_0 l >> 1$. Assuming that the TM modes, Eqs. (5), are of the form

$$\mathcal{E}_i(x,z,t) = G_i(\zeta,k_0)\ \exp\ [i(k_0 l\ W(x,z) - \omega t)],\ i = x,\ z,\ y, \quad (10)$$

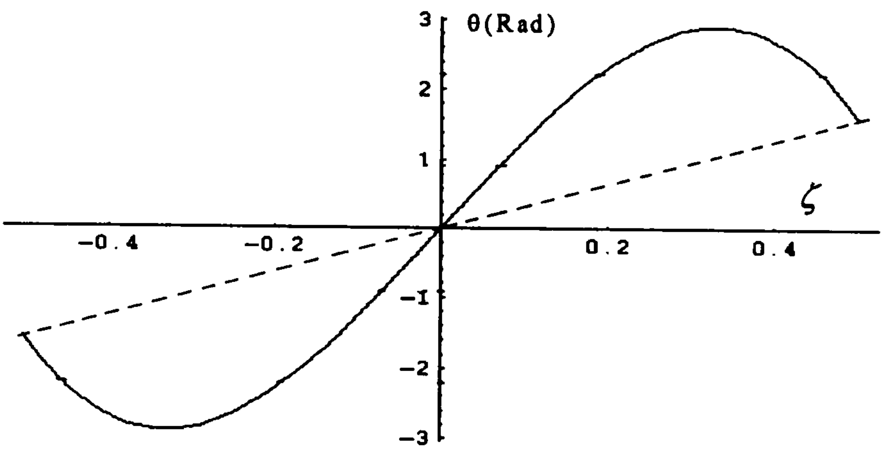

FIGURE 2 The orientational configuration in the low intensity limit with hydrodynamic effects

where $W(\mathbf{r})$ is the characteristic function of Hamilton which is equal to the difference in optical paths of a ray propagating between two points in the cell. However, in this limit it is not necessary to calculate the TM modes explicitly, although it can be done. Instead, we shall calculate the ray trajectories from the corresponding eikonal equation. In terms of the variable $S(\zeta)$ defined by $W(\zeta,\chi) = p\chi + S(\zeta)$, with $\chi \equiv x/l$, this equation (Hamilton-Jacobi) turns out to be the following ordinary differential equation

$$\varepsilon_{zz}(dS(\zeta)/d\zeta)^2 + 2p\varepsilon_{xz}\ dS(\zeta)/d\zeta + p^2\varepsilon_{xx} - \varepsilon_{\parallel}\varepsilon_{\perp} = 0, \quad (11)$$

whose general solution reads

$$S_{\pm}^{(0)}(\zeta) = \pm \int_0^{\zeta} d\zeta' [p\varepsilon_{xz} \pm (\varepsilon_{\parallel}\varepsilon_{\perp}(\varepsilon_{zz} - p^2))^{1/2}]/\varepsilon_{zz} + S_0^{\pm}. \quad (12)$$

The superscript $^{(0)}$ indicates that $S(\zeta)$ has been calculated to order zero in $1/k_0 l$, that is, in the optical limit. $S_0^{\pm}$ are integration constants whose specific values can be determined from the boundary conditions that the TM modes should satisfy at the metallic plates. The ray trajectories $\gamma = \gamma(\zeta,\chi)$ are defined by $\gamma = \partial W(\zeta,\chi)/\partial p$, where γ is the generalized coordinate conjugated to the momentum p. From the definition of W in terms of $S(\zeta)$ and Eq. (12) we get that

$$\gamma = \chi - \int_0^{\zeta} d\eta \, \{\varepsilon_{xz} \pm p \, \varepsilon_{zz}^{-1} \, [\varepsilon_{\parallel}\varepsilon_{\perp}/(\varepsilon_{zz}-p^2)]^{1/2}\}. \tag{13}$$

The parameter p may be expressed in terms of the propagation angle α by $p = (\varepsilon_{\parallel})^{1/2} \, [1 + (\varepsilon_{\perp}/\varepsilon_{\parallel}) \cot^2 \alpha]^{-1/2}$. Since the components of ε_{ij} are functions of θ, substitution of the solution (9), which depends on the flow through the parameter N, leads to explicit expressions for the ray trajectories in the presence of hydrodynamic flow. We do not give this analytic expression but instead we calculate an explicit trajectory. However, also note that Eq.(13) is real only if $\varepsilon_{zz} < p^2$, which means that the ray can only propagate in some regions within the cell; that is, there exist caustics. This behaviour is illustrated in Fig.3 for MBBA. Three trajectories are shown for a propagation angle $\alpha = 45^0$ with $R = 10^{-2}$ and N>0. The curves show that in contrast to the results in I where the trajectories were all concentrated in the central part of the cell, the presence of flow confines them into well defined independent stripes whose number, width and position within the cell is determined both, by flow and material parameters. As R increases and the flow becomes more intense, the number of stripes also increases.

WKB LIMIT

Since the optical limit, $(\omega l/c) \gg 1$, holds true for high frequencies of the beam within the cell, it would be of interest to find out if the wave guiding effect still exists for lower frequencies. To this end it is necessary to take the next succesive approximation to the optical limit and keep now terms of order $1/k_0 l$. This defines the WKB limit. Since in this limit the concept of ray trajectory looses its meaning, to exhibit the existence of the wave guiding effect we first calculate the TM modes up to first order in $1/k_0 l$ and then show that the electromagnetic energy density distributes itself around the central part of the cell. To this order Eq.(11) now is

$$\varepsilon_{zz}[dS(\zeta)/d\zeta]^2 + 2p\varepsilon_{xz}\, dS(\zeta)/d\zeta + p^2\varepsilon_{xx} - \varepsilon_{\parallel}\varepsilon_{\perp} =$$

$$(1/k_0 l)\ [\varepsilon_{zz} d^2S_{\pm}^{(0)}(\zeta)/d\zeta^2 + d\varepsilon_{zz}/d\zeta + pd\varepsilon_{xz}/d\zeta]. \qquad (14)$$

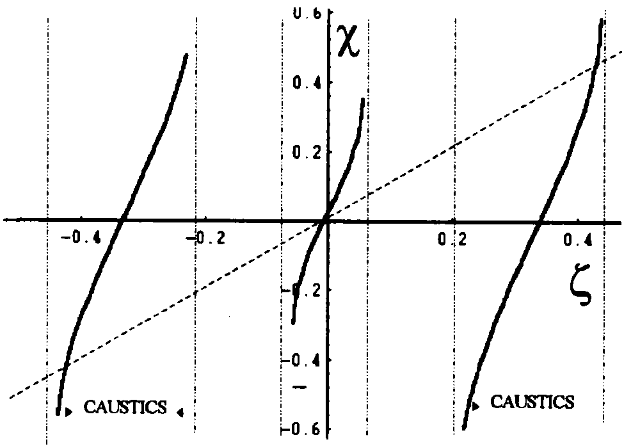

FIGURE 3 Ray trajectories for MBBA at $T = 25^{0}C$. $\alpha = 45^{0}$, $R = 10^{-2}$ and $N>0$.

From Eqs.(12) and (14) we find that

$$S(\zeta) = S_{\pm}^{(0)}(\zeta) + (1/2k_0 l)\ \ln[\varepsilon_{\parallel}\varepsilon_{\perp}(\varepsilon_{zz}-p^2)]^{1/2} + C_{\pm} + O(1/k_0 l)^2. (15)$$

Substitution of this expression into Eq.(10) and using the relation between $W(\chi,\zeta)$ and $S(\zeta)$, we can obtain explicit expressions for the TM modes in the WKB limit. It should be pointed out that in doing so the coupling between the orientation and the optical field, as measured by q, is entirely neglected. To order zero in q the explicit forms of these modes and the integration constants $C_{\pm}$ in Eq.(15), are obtained by satisfying the appropriate boundary conditions for TM modes at the metallic plates and by fulfilling the conection formulas at the corresponding caustics. [9] The resulting equations are too complicated to give them here, however, their explicit form can be found in reference 9. Using these resulting expressions we calculate the distribution of the electromagnetic energy density, $u_{em}=(1/8\pi)[\varepsilon_{\perp}\mathbf{E}^2 + \varepsilon_a(\mathbf{n}\cdot\mathbf{E})^2 + H_y^2]$, within the cell. For the same system and parameters as in Fig.3, we obtain the curves plotted in Fig.4. Note that most of the electromagnetic energy density is indeed concentrated around the central part of the cell; this shows the existence of the wave guiding effect even in the WKB limit and in the presence of hydrodynamic flows.

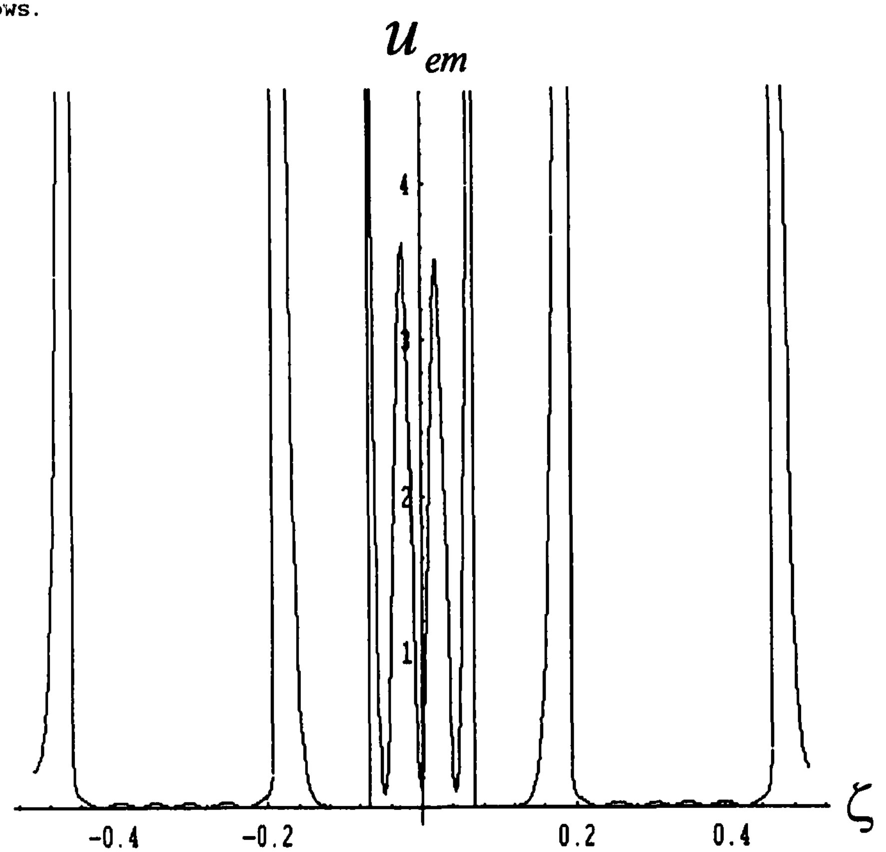

FIGURE 4 Distribution of the electromagnetic energy density u_{em} as a function of ζ. Same parameters as in Fig.3.

CONCLUSIONS

In this work we have shown that for the nematic cell model that we have introduced in I, a wave guiding effect still exists in the presence of both, internal backflows and externally induced stationary flows. In contrast to the situation discussed in I where hydrodynamic flows were entirely neglected, here their effect is manifested in the optical limit by confining the trajectories of the optical field into independent stripes whose number, position and width is determined by the Reynolds number and the material parameters. Another feature of our model that should be emphasized is that in our discussion the coupling between the orientational dynamics and the electromagnetic field, as measured by the parameter q, was entirely neglected. When this restriction is relaxed, the consideration of higher orders in q makes nonlinear the dynamics of both, the reorientation angle θ and the TM modes. This is a richer physical situation and it can be shown, for instance, that for a wave packet of normal modes for which the wave guiding effect holds individually, the field propagates as optical solitons concentrated along the central part of the cell.[10] However, whether the presence of flow reinforces or destroys the wave guiding effect in this case remains to be assessed.

ACKNOWLEDGEMENT

This research was partially supported by DGAPA-UNAM, Project IN100494

REFERENCES

1. B. Ya. Zel'dovich, N.F. Pilipotskii, A. V. Sulchov and N. V. Tabiryan, JETP Lett., 31, 363 (1980)
2. N. Y. Tabiryan, A. V. Sukhov and B. Ya Zel'dovich, Mol. Cryst. Liq. Cryst. 136, 1 (1986)
3. S. D. Durbin, S. M. Arakelian and Y. R. Shen, Phys. Rev. Lett. 47, 1411 (1981)
4. E. Santamoto, G. Abatte, P. Maddalena and L. Marrucci, Phys. Rev. Lett. 64, 1377 (1990)
5. H. Lin and P. Palffy-Muhoray, Liquid Crystals 14, 1977 (1993)
6. P. V. Vidakovic, M. Coquilay and F. Salin, J. Opt. Soc. Am. B, 4, 998 (1987)
7. J. A. Reyes and R. F. Rodríguez, J. Nonlin. Opt. Prop. Mat. 4, xxx(1995) (to be published)
8. M. San Miguel and F. Sagués, Phys. Rev. A 36, 1883 (1987)
9. J. A. Reyes, Ph.D. Dissertation, Universidad Nacional Autónoma de México, 1995 (in Spanish)
10. R. F. Rodríguez and J. A. Reyes (1995) (unpublished)